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A surveillance-interdiction model for remote area operations

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A SURVEILLANCE-INTERDICTION MODEL FOR REMOTE AREA OPERATIONS

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THESIS

A SURVEILLANCE-INTERDICTION MODEL

FOR REMOTE AREA OPERATIONS

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A Surveillance-Interdiction Model

For Remote Area Operations

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ABSTRACT

This thesis presents a mathematical model of conducting limited scale surveillance-interdiction operations in a remote area environment. The model, which is stochastic in nature, provides CI patrol schedules in such a manner as to minimize the patrol effort expended, subject to various resource and operational constraints established by the local CI commander. Patrol schedules are developed so as to minimize the ability of the insurgent force to accurately predict future CI operational plans. In addition to generating patrol schedules, for a specified patrol plan, the model predicts additional information such as the expected number of active CI patrols at any given time, the expected duration of a CI patrol, as well as an estimate of the insurgent infiltration rate along a given segment of the infiltration route. Finally, the model is capable of providing probabilistic statements concerning the adequacy of a specified force level to satisfy the requirements of a given patrol plan.

TABLE OF CONTENTS

LIST OF FIGURES

I. INTRODUCTION

A. DISCUSSION OF INFILTRATION

Few, if any, successful insurgent movements have been totally, or even primarily, internally supported, politically or militarily. For such a movement to successfully accomplish its objective of political and social revolution, it is necessary, in particular during the later, more intensive phases, to receive outside support from some sponsoring power. Such support, normally in the form of men and material, is generally infiltrated into the target country covertly by insurgent forces over a complex system of infiltration routes.

During the early stages of an insurgency, the infiltration effort is generally limited in scope, since initially, overt insurgent military operations are necessarily restricted to preclude prematurely alarming the existing government. In addition, initially a significant percentage of insurgent support is obtained from that segment of the local population sympathetic to the insurgent objectives.

However, as the insurgent movement progresses, reliance increases on the support infiltrated into the target country. This is due to increased requirements created by more extensive insurgent operations, losses of men and supplies to the more active counter-insurgent operations, as well as a decrease in the amount of support

received from the local population due to increased CI efforts in population and resource control.

In many insurgent movements, external support is infiltrated to the insurgent organization from a contigious political sanctuary. By necessity, the routes along which this infiltration takes place, are located in terrain which affords maximum security to the infiltration operation. If the terrain is generally flat, open, and with limited over-head cover, then the corridors are usually characterized by a large number of not necessarily well defined infiltration routes, e. g., the intricate canal systems of the IV Corps Tactical Zone of SVN. On the other hand, if the terrain is mountainous, heavily vegetated with abundant over-head cover, then there are usually only a limited number of suitable routes available for movement of large amounts of supplies and men, e. g., the I CTZ of SVN.

It should be noted, that in general, the CI forces are familiar only with the location of infiltration corridors, which themselves are seldom well defined. These corridors consist of numerous individual trails of various capacities which allow the insurgent great flexibility in conducting the infiltration effort. The complexity of any infiltration corridor, will naturally depend on many factors, a few of which were mentioned above.

B. CONDUCT OF INFILTRATION

In planning and conducting the infiltration program, the insurgent is interested in affecting the movement of men and material along those routes which offer the maximum security to the operation. Essentially, this means along those routes which provide for a minimum probability of being detected by the CI force. This implies, that to a degree, the success with which the insurgent conducts infiltration through an area may, in part, be expressed as a function of the insurgents knowledge about current and proposed CI surveillance-interdiction operations. Obviously, the ideal situation from the insurgents point of view would be to have complete information concerning CI actions and intentions. This would allow the insurgent to conduct infiltration operations in those areas void of CI interference, thus reducing the probability of detection, and hence expected insurgent operational losses. On the other hand, the worst situation the insurgent could face, would be that of having no information concerning current and future CI operations. In this situation, the insurgent would have no such "reliable" plan upon which to base the infiltration schedule, and would be forced to allocate the infiltration effort piecemeal along many routes in hopes of successfully delivering an acceptable portion of the overall requirement. This last situation represents a case of "maximum cost" to the insurgent, while providing the CI with the best opportunity to obtain information on insurgent infiltration efforts.

C. INTERDICTION OPERATIONS

Historically, possibly with the exception of the Malayian insurgency, it has been demonstrated that stopping infiltration is a very difficult (if not impossible) and costly task. These interdiction operations generally include the combined efforts of ground, air, and if appropriate, water-borne elements. This paper will primarily restrict itself to the investigation of the ground effort against insurgent infiltration, in the area adjacent to the common border between a contigious political sanctuary and the target country. However, as will be evident later in this paper, the development could certainly be translated into the air or water-borne environment. In particular, this paper will focus on the border surveillance-interdiction effort of those military/para-military camps deployed along the border described above.

The mission of these remote camps is to establish a surveillance screen along the border, in an effort to develop information concerning insurgent infiltration operations, as well as the conduct of limited scale offensive ground operations against infiltration when identified. These offensive operations are deliberately limited in scale and complexity because of the generally limited capability of the forces being employed. It might be said that the primary purpose of these installations is simply to make the infiltration process as costly as possible to the insurgent.

To accomplish the border surveillance-interdiction mission, these forces have utilized techniques varying from the stationary "trail watcher" to the larger, more mobile reconnaissance patrols which range throughout the camps tactical area of responsibility (TAOR). The former technique, which relies heavily on small teams of three to five individuals, positioned along primary infiltration routes to report observed activity, is generally employed only by highly trained, well-equipped military organizations. The latter technique, which places less demand on individual qualifications and equipment, is normally employed by the remote camps of the type addressed in this paper.

The problem faced by the local CI commander is how best to employ his limited resources so as to (1) maximize the cost of infiltration to the insurgent, and (2) maximize the amount of useful intelligence developed by CI ground patrols deployed in the camps TAOR, subject to various constraints.

D. GENERAL SITUATION

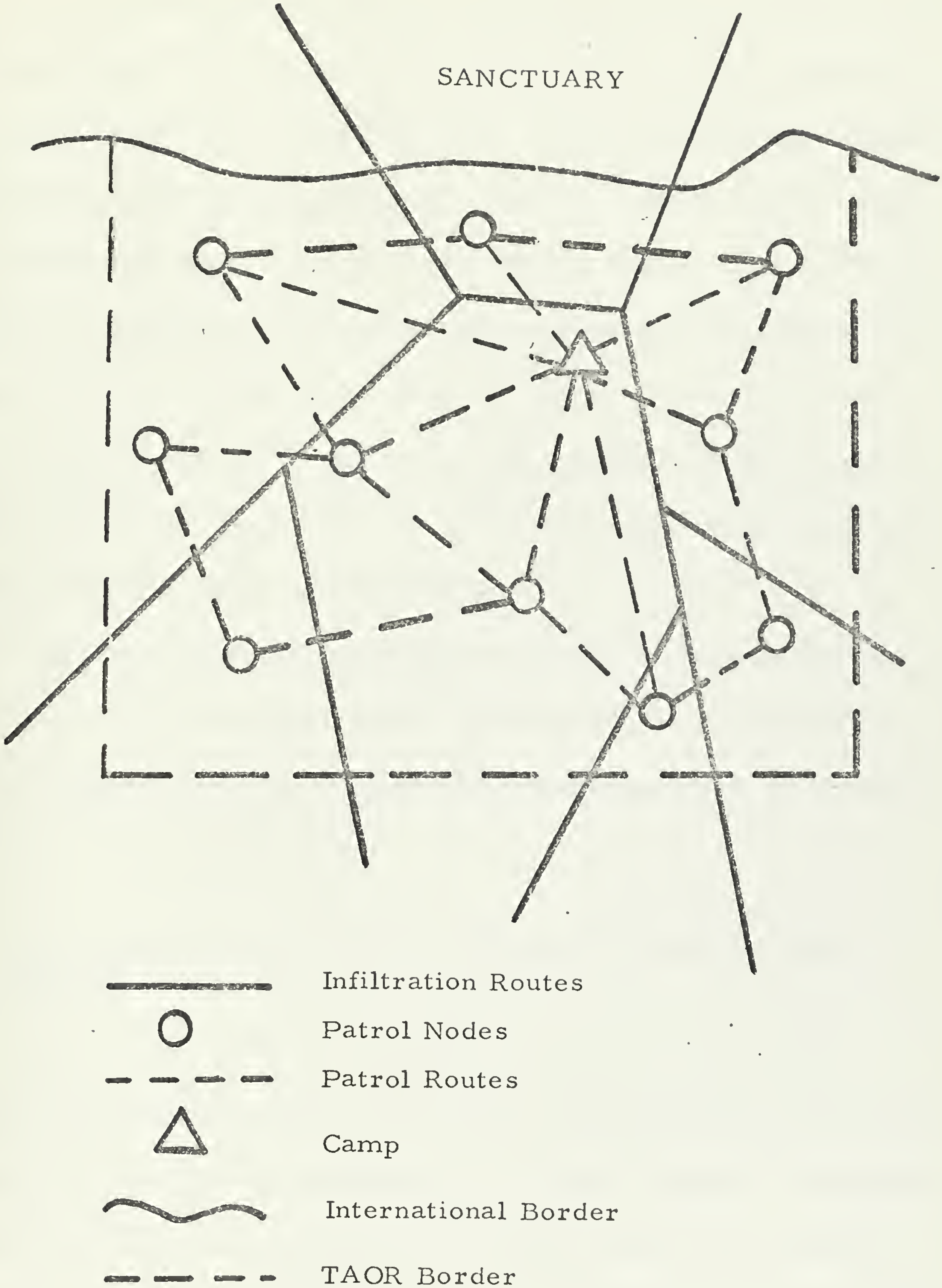
For the purposes of this paper, it will be assumed that the CI force has been assigned a TAOR contigious with the border of an adjacent political sanctuary. The CI force has been assigned the mission of border surveillance and interdiction. Among the CI's primary objectives are:

1. the conduct of surveillance of that portion of the border within its TAOR to develop intelligence concerning insurgent infiltration operations.

2. the conduct of limited scale ground operations within the TAOR to interdict insurgent infiltration operations.

Figure 1 illustrates a typical TAOR. It is assumed that the insurgent has sufficient experience in the operational area to identify the major trail complexes used by the insurgent for infiltration purposes. Such a hypothetical trail network appears in the figure. It should be noted, that the trail system shown in the figure represents but a small segment of the overall infiltration corridor which this particular camp has been assigned the mission of surveilling. Adjacent camps are assigned TAOR's, which accumulatively, span the entire width of the corridor. Note that the infiltration trails are mutually supporting, to provide the insurgent with important flexibility in directing the infiltration operation.

To maintain ground surveillance along the border, as well as along the existing trails within the TAOR, the CI force commander conducts ground reconnaissance patrols throughout the operational area. The number of patrols in the field at any time will necessarily depend on many factors, including, the size of the TAOR, the strength of the camp, the nature of the trail complexes, as well as the current level of insurgent activity.



THE OPERATIONAL AREA

Figure 1.

It is assumed that the CI force commander has developed, or has the capability of developing, a set of patrol routes to be used by the CI ground operations. Such routes are designed to provide maximum surveillance coverage to previously identified trails, while allowing for periodic reconnaissance of other critical areas within the TAOR. The latter capability allows for the identification of new routes which may be built by the insurgent, as older routes become too costly to use. It should be noted, that these patrol routes will not coincide with existing insurgent trails, but will merely allow for adequate surveillance of activity along these trails.

A hypothetical patrol plan for the assumed TAOR is contained in Figure 1. It must be realized that periodically, and for many reasons, the CI commander will re-design the camps patrol plan to account for changes in the local situation.

E. AN APPROACH TO THE SURVEILLANCE-INTERDICTION PROBLEM

Assume that a CI force, assigned the border surveillance-interdiction mission, utilizes the mobile reconnaissance patrol as its primary vehicle for accomplishing the assigned mission. As stated earlier, much of the question of surveillance-interdiction may be reduced to the problem of the CI force precluding the insurgent from gaining information concerning current and future operations. That is, if the insurgent is unable to accurately predict the location of future CI efforts, then the insurgent infiltration process becomes

quite risky, and hence costly. It would then appear to be to the CI's advantage to develop a program which would reduce the insurgents capability of predicting this information. A scheme which certainly satisfies this requirement is a "random" patrolling system. Specifically, if the CI patrol schedule, with regards to both time and location, were truly random, it would be impossible for the insurgent to predict, based on previous experience, the proposed intentions of the CI force. While the insurgent is most interested in being able to predict accurately when and where CI patrols will be at any given time, the CI commander is interested in denying the insurgent this very information.

In view of the above, the objective of this paper is to develop a patrol system which possesses the quality of truly being "as random as possible" over those areas of primary interest to the CI organization. Such a model, possessing this desired random characteristic, is developed in Section II.

II. MODEL DEVELOPMENT

A. INTRODUCTION TO THE MODEL

The basis for the model developed in this paper is taken from work done by Rosenshine [1], which deals with the theory of patrol scheduling. The model developed by Rosenshine, which concerned police patrol scheduling, has been modified for the counter-insurgency situation.

B. OBJECTIVES OF THE MATHEMATICAL MODEL

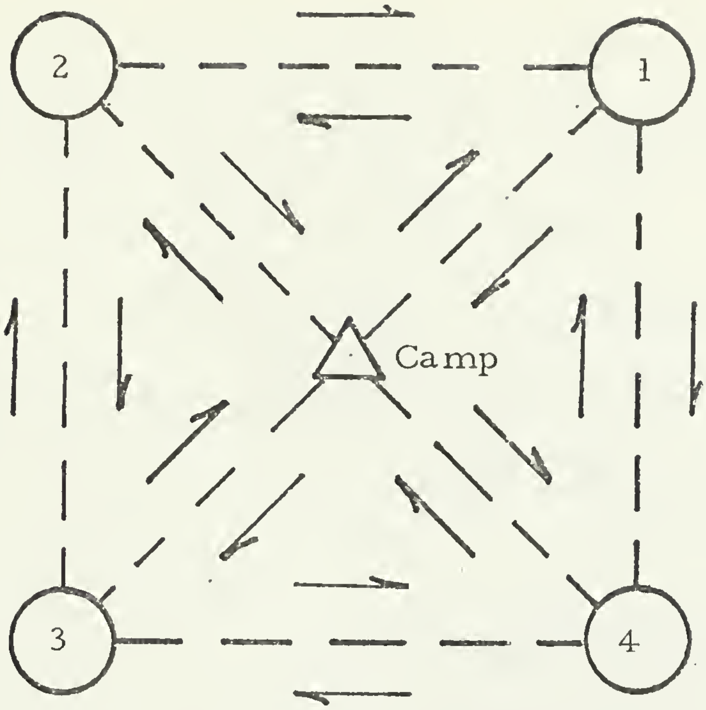
The objectives of the model presented in this section are to:

1. determine the appropriate inter-departure times for patrols dispatched by a CI force.
2. identify the optimal average patrol rates along the various segments of the patrol route network, and,
3. define the appropriate route to be utilized by a particular patrol.

These three objectives are to be accomplished in such a manner as to provide "as random as possible" coverage to those areas of primary interest to the CI commander.

C. PATROL INTER-DEPARTURE TIMES

In the development of the basic model for this paper, the CI patrol route plan described in Figure 2, will be utilized for simplicity.



THE PATROL NETWORK
Figure 2.

It is assumed that all route segments in Figure 2. may be traveled in either direction. In addition, no restriction will be placed on the equality of the length of the various route segments, either in actual distance covered, or time required to traverse the segment. It should be recalled, that the primary objective of the model is to provide for adequate patrolling throughout the network on as random a basis as possible. By "as random as possible" is meant, that if an insurgent was to be stationed at some location in the patrol network recording the times at which CI patrols passed that location, the insurgents ability to predict the arrival of the next patrol, based on previous arrival data which had been collected, would be no better than an estimate based on no historical data. That is, the insurgents ability to predict the arrival time of the next patrol would not be improved

by prior knowledge concerning previous patrols. More rigorously, if X_i defines the inter-arrival time between the i^{th} and the $(i-1)^{st}$ patrols, then, "as random as possible" implies:

$$P(X_{n+1} = t/x_1, x_2, \ldots x_n) = P(X_{n+1} = t)$$

where P(A) represents the probability that event A occurs. Such a "memoryless" condition certainly suggests the use of a scheme based on an exponential distribution.

As an example, assume that the camp patrol plan contained only one route, which consisted of only one segment. Such a situation is shown in Figure 3.

A SIMPLIFIED PATROL NETWORK

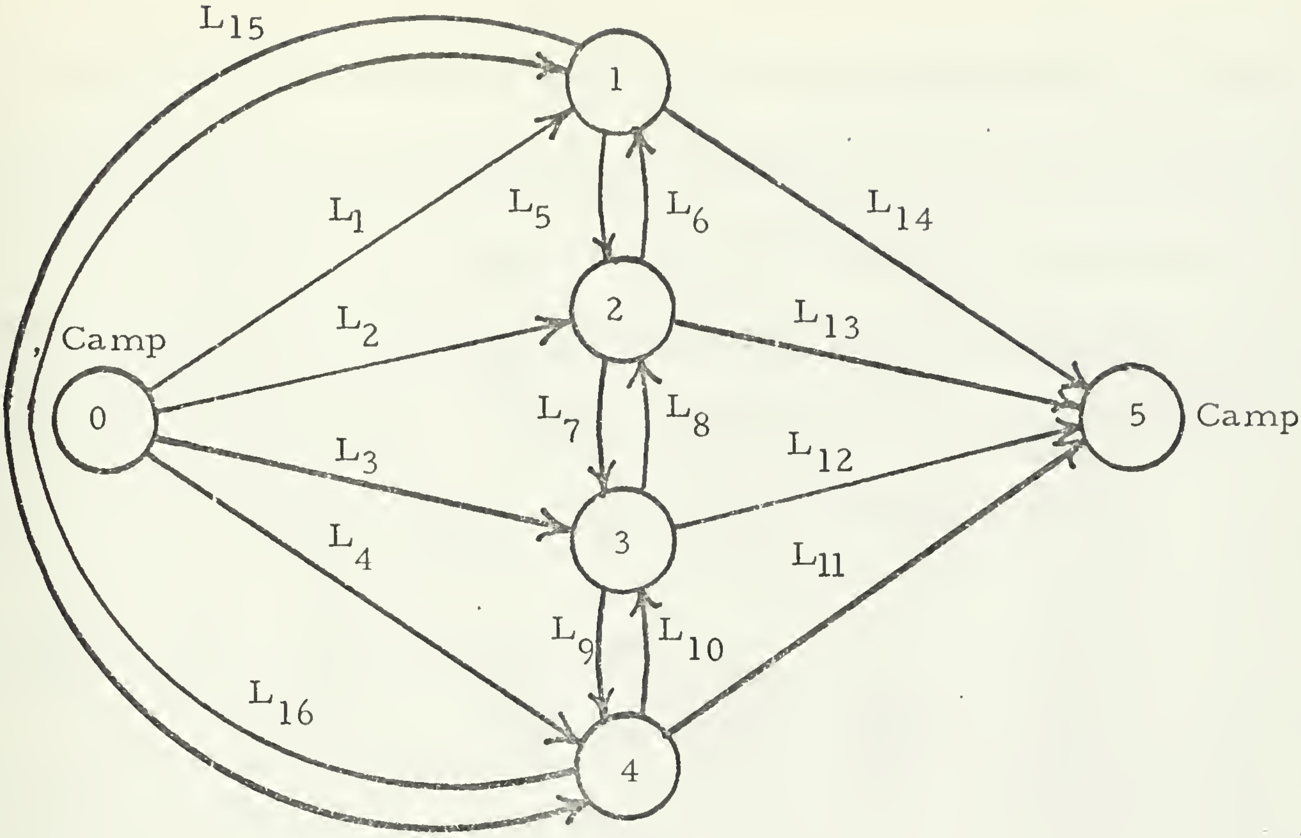
Figure 3.

Developing a patrol scheme for the route network described in Figure 3, which would possess the desired random qualities, could be accomplished by simply dispatching patrols from 1 with inter-departure times determined from an exponential density function:

$$f_T(t) = Le^{-Lt} \qquad \text{for } t \geq 0$$

where L is the (average) rate at which patrols are dispatched along the route, and "t" is the time between successive patrols.

Now consider a network with multiple routes and route segments, such as in Figure 2. Figure 2 is a planar graph, and isomorphic to the network described in Figure 4, [5].



AN ISOMORPHIC EQUIVALENCE TO THE PATROL NETWORK

Figure 4.

In the situation described in Figure 4., CI patrols would be dispatched from the camp, or source, at a rate $L_T = L_1 + L_2 + L_3 + L_4$. The inter-departure times between patrols, t, would be determined as before from the probability density function:

$$f_T(t) = L_T e^{-L_T t} \quad t \geq 0$$

where L_T and t are defined as above.

D. ARC AVERAGE PATROL RATES

Now suppose that the CI force commander, based on experience in the operational area, determined that, as a minimum, the i^{th} arc of the patrol plan must be surveilled at an average rate L_i', in order to accomplish the camp's assigned mission. That is:

$$L_i \geq L_i' \quad \text{for all i}$$

In addition, the commander determines the average time, t_i, required for a patrol to traverse the i^{th} arc of the network. The problem then is to identify the actual rates, L_i, which will minimize the total patrol time expended by the camp, subject to the $L_i^{\prime}$ and t_i constraints which have been established. Utilizing Figures 2 and 4, the problem is to

$$\text{Minimize} \sum_{1}^{n} L_i \; t_i \text{ (where } n = 16 \text{ in Figure 4)}$$

subject to the following constraints:

$$L_i \geq L_i^{\prime} \;\; 0 \quad \text{for all } i = 1, 2, \ldots, 16$$

node 1 $\quad L_1+L_6+L_{15}-L_5-L_{14}-L_{16}=0$

node 2 $\quad L_2+L_5+L_8-L_6-L_7-L_{13} \quad =0$

node 3 $\quad L_3+L_7+L_{10}-L_8-L_9-L_{12} \quad =0$

node 4 $\quad L_4+L_9+L_{16}-L_{10}-L_{11}-L_{15}=0$

node 0 $\quad L_1+L_2+L_3+L_4 \leq La$ (upper limit for availability of patrols)

node 5 $\quad L_{11}+L_{12}+L_{13}+L_{14} \leq La$ (upper limit for availability of patrols)

$L_1+L_2+L_3+L_4-L_{11}-L_{12}-L_{13}-L_{14} = 0$ (equality of source output and sink input)

The solution to the above linear programming problem will identify the appropriate Li necessary to minimize the overall CI patrol effort subject to the established constraints [5].

E. ROUTE SELECTION

Having identified the optimal patrol rates along the n assumed route segments, it is now necessary to determine the actual route to be utilized by a particular patrol. Specifically, it is necessary to

identify those nodes within the network in Figure 4, through which a given patrol will pass during the conduct of an operation. It must be remembered that the L_i were established to insure that the scheme developed would take into account the desires of the commander pertaining to the frequency with which a particular route segment is surveilled. Therefore, the procedure selected should in some manner depend on the relative values of the L_i. That is, assuming that the patrol has reached node k, the decision as to which arc should be used when departing k should depend on the relative values of the rates, L_j, $j=i, 2, \ldots, m$, for all m possible arcs departing node k. If we define P_j to be the probability that the J^{th} arc is utilized when departing node k, then,

$$P_j = \frac{L_j}{\sum_{j=1}^{m} L_j} \qquad j=1, 2, \ldots, m$$

represents a probability distribution function, pdf, for the k^{th} node [2]. Obviously each node may possess a unique pdf of the type described above. Utilizing this pdf, an appropriate decision may be made at each node to determine the optimal path to be followed by the patrol when departing that node.

It should be noted, that the use of a linear program to determine the optimal patrol rates, a well defined probability density function to identify patrol inter-departure times, and the above pdf, to determine the proper sequence of nodes to be visited by patrols, necessitates

the use of a computer program to generate large numbers of patrol schedules.

F. THE EXPECTED NUMBER OF ACTIVE PATROLS AT TIME "T"

Another question of prime importance to the CI commander is the average number of patrols which will be in the network at some time t. This information is of great value when planning for maintenance, training, or simply establishing troop level requirements.

Recalling that the inter-departure time for patrols is exponentially distributed, with parameter L_T, it follows that the introduction of patrols into the network represents a Poisson renewal process. The number of patrols which enter the network during the time interval (O, T), denoted N(T), is a Poisson random variable, with parameter (L_T T), where,

$$L_T = \sum_{i=1}^{m} L_i \qquad i = 1, 2, \ldots, m$$

and there are m different route segments available for use by patrols when departing the camp, or source. As indicated earlier, in Figure 4, $L_T = L_1 + L_2 + L_3 + L_4$.

Define X_T to be the total number of patrols active in the network at time T, given that a total of N(T) = n patrols actually entered the network during the interval (0, T). Therefore, the total probability law states,

$$P(X_T = k) = \sum_{n=o}^{\infty} P(X_T = k / N(T) = n) \; P(N(T) = n)$$

Define:

$Xi = 1$ if the i^{th} patrol is still in network at time T.

$Xi = o$ if the i^{th} patrol is not in the network at time T.

$p = P$ (the i^{th} patrol is in the network at time T/ that the i^{th} patrol originally entered the network)

Then Xi is a Bernoulli random variable with parameter p.

Then,

$$X_T = \sum_{i=1}^{n} Xi$$

and X_T is a Binomial random variable with parameters p and n.

Therefore,

$$P(X_T=k) = \binom{n}{k} p^k (1-p)^{n-k} \qquad k=0, 1, \ldots, n$$

Since N(T) is a Poisson distributed random variable, with parameter $(L_T T)$, then

$$P(X_T=k) = \sum_{n=o}^{\infty} \frac{\binom{n}{k} p^k (1-p)^{n-k} (L_T T)^n e^{-L_T T}}{n!}$$

$$= \frac{(L_T pT)^k e^{-L_T pT}}{k!} \qquad k=0, 1, 2, \ldots\ldots$$

and the average number of patrols active in the network at time T is:

$$E(X_T) = L_T pT$$

The steady state distribution for the number of patrols in the network at time T may be obtained in the following manner. The expected number of patrols in the system at any point in time is the product

of the rate at which patrols are introduced into the network and the average time a patrol remains in the network. Then,

$$\lim_{T \to \infty} L_T p T = L_T E(T_s)$$

To evaluate the $E(T_s)$, recall that the route determination decision was a stochastic process. Accordingly, the patrol network may be represented by an imbedded Markov Chain. The source, or camp, is represented by a reflecting barrier; the sink by an absorbing barrier. For the patrol network represented in Figure 4, the transition matrix for the Markov Chain is as follows.

State j

State i	0	1	2	3	4	5
0	0	$\frac{L_1}{L_1+L_2+L_3+L_4}$	$\frac{L_2}{L_1+L_2+L_3+L_4}$	$\frac{L_3}{L_1+L_2+L_3+L_4}$	$\frac{L_4}{L_1+L_2+L_3+L_4}$	0
1	0	0	$\frac{L_5}{L_5+L_{14}+L_{16}}$	0	$\frac{L_{14}}{L_5+L_{14}+L_{16}}$	$\frac{L_{16}}{L_5+L_{14}+L_{16}}$
2	0	$\frac{L_6}{L_6+L_7+L_{13}}$	0	$\frac{L_7}{L_6+L_7+L_{13}}$	0	$\frac{L_{13}}{L_6+L_7+L_{13}}$
3	0.	0	$\frac{L_8}{L_8+L_9+L_{12}}$	0	$\frac{L_9}{L_8+L_9+L_{12}}$	$\frac{L_{12}}{L_8+L_9+L_{12}}$
4	0	$\frac{L_{15}}{L_{10}+L_{11}+L_{15}}$	0	$\frac{L_{10}}{L_{10}+L_{11}+L_{15}}$	0	$\frac{L_{11}}{L_{10}+L_{11}+L_{15}}$
5	0	0	0	0	0	1

MARKOV TRANSITION MATRIX FOR A PATROL NETWORK

Figure 5.

In Figure 5, the $(i, j)^{th}$ entry represents the probability, $P_{i,j}$, that when departing the i^{th} node the patrol will proceed to the J^{th} node. Each node in the network represents a state of the Markov Chain, and

$$P_{i,j} = 0 \quad \text{if the } j^{th} \text{ state is not accessible from the } i^{th} \text{ state}$$

$$= \frac{L_{i,j}}{\sum_{k=1}^{m} L_{i,k}} \qquad k=1, 2, \ldots, m$$

where $L_{i,j}$ is the patrol rate between the i^{th} and j^{th} nodes, and the summation in the denominator is over all m routes available for departing the i^{th} node.

Define $m_{i,j}$ to be the mean number of transitions till absorption at the j^{th} state for a Markov Chain which starts in state i. That is, $m_{i,j}$ represents the mean number of route-segments traversed by a patrol during an operation which starts in state i, and terminates in state j. The following expression for $m_{i,j}$ is developed by Hillier and Lieberman in reference (2).

$$m_{i,j} = 1 + \sum_{k \neq j} P_{i,k} m_{k,j}$$

Then $m_{o,5}$ in Figure 4, which is the expected number of transitions till absorption at state 5 for the Markov chain in Figure 5, assuming the system was initially in state 0, may be determined from the solution to the system of equations generated by the above relationship.

If we define t_{ij} to be the average time required to traverse the arc between the i^{th} and j^{th} nodes, then

$$m_{i,j} = t_{ij} + \sum_{k \neq j} P_{i,k} \quad m_{k,j}$$

where $m_{i,j}$ is now the average time required for a patrol to travel from the i^{th} node to the j^{th} node. In particular, $E(T_s) = m_{i,j}$, for a patrol starting in state i and terminating in state j may be determined from a solution to this system of equations.

In view of the above, the steady-state distribution of the number of patrols in the network at time T may be expressed as

$$P(X_T = k) = \frac{e^{-L_T E(T_s)} (L_T \, E(T_s))^k}{k!}$$

which is a poisson distribution, with parameters $L_T E(T_s)$.

G. APPLICATION OF THE EXPECTED VALUE RELATIONSHIPS

During the planning phase for the establishment of remote military/para-military installations of the type discussed in this paper, it is often necessary for the planning staff to identify the most efficient size force with which to garrison the camp. The model presented thus far is ideally suited for developing estimates of this nature.

Assume that the planning staff has selected an initial TAOR to be occupied by the proposed camp. Based on existing intelligence, the planners develop an initial patrol network for the area. Estimates are

established for the required arc patrol rates, L_i', as well as the expected time, t_i, required to traverse each arc. Based on this initial representation of the operation, the appropriate LP problem is solved to identify the actual arc patrol rates. In addition, the Markovian relationships discussed in the preceeding paragraph are utilized to determine $E(T_s)$. Since it was previously demonstrated that the number of active patrols in the network is distributed Poisson with parameter $L_T E(T_s)$, for steady-state conditions, then it can be stated that,

$$P(X_T \leq K) = \sum_{j=0}^{K} \frac{(L_T E(T_s))^j \; e^{-L_T E(T_s)}}{j\,!}$$

If k is defined to represent the number of active patrols which could be supported by a camp force of a given level, then the probability that the number of patrols required in the field will not exceed the availability of patrol effort is defined as above. In this manner, planners are able to develop force estimates necessary to insure, for some specified probability level, that an adequate force will be available to support the proposed operational plan. For a specified probability level, values of K may be obtained for different combinations of L_T and $E(T_s)$. An analysis of how K varies as a function of the proposed patrol plans, and hence L_T and $E(T_s)$, can be an effective aid to the decision maker when selecting the appropriate force level to be assigned to the TAOR under consideration.

If on the other hand, assume planners are provided a lower limit on the acceptable probability, P_L, of having adequate patrol effort to support the camp mission, and asked to determine the appropriate force level for a given patrol network, then, the K, which satisfies,

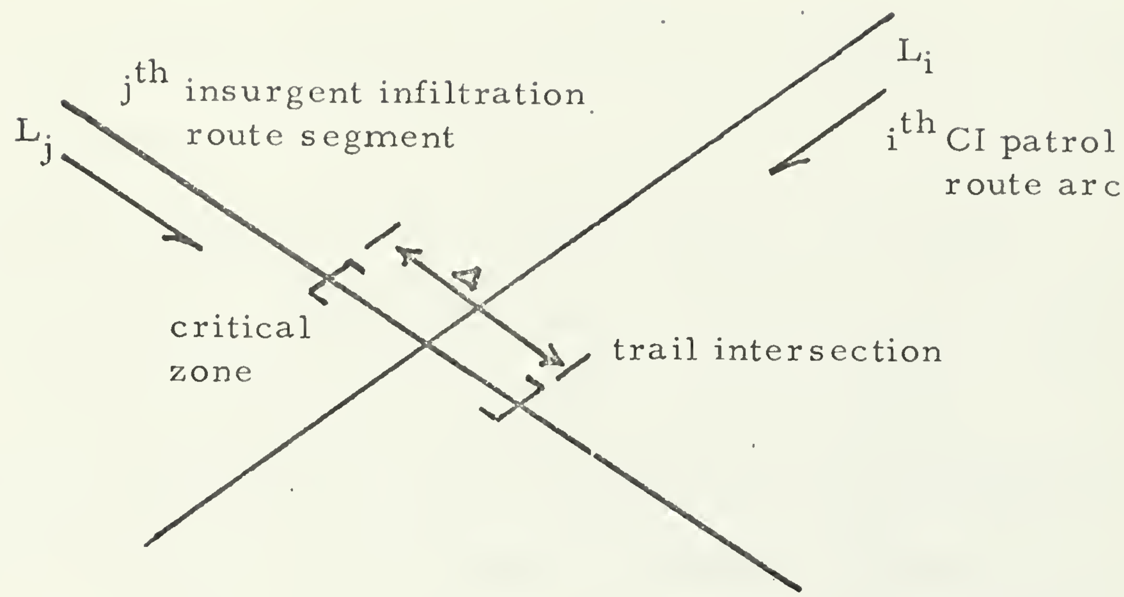
$$P_L = \sum_{J=0}^{K} \frac{(L_T E(T_s))^j \, e^{-L_T E(T_s)}}{j!}$$

will represent the required patrol effort. This value may then be translated into a meaningful force level recommendation.

H. ESTIMATING THE INFILTRATION RATE

A primary mission of the remote military/para-military camp is to develop meaningful information concerning insurgent infiltration operations. Among the principal items of interest to the CI force is the rate at which insurgent forces conduct infiltration through the assigned TAOR. The model described in this paper provides a means of developing an approximation for the infiltration rate along given route segments. This section will describe the approximating procedure.

Assume that Figure 6 describes the intersection of a CI patrol route arc and an insurgent infiltration trail segment.



CI AND INSURGENT TRAIL INTERSECTION

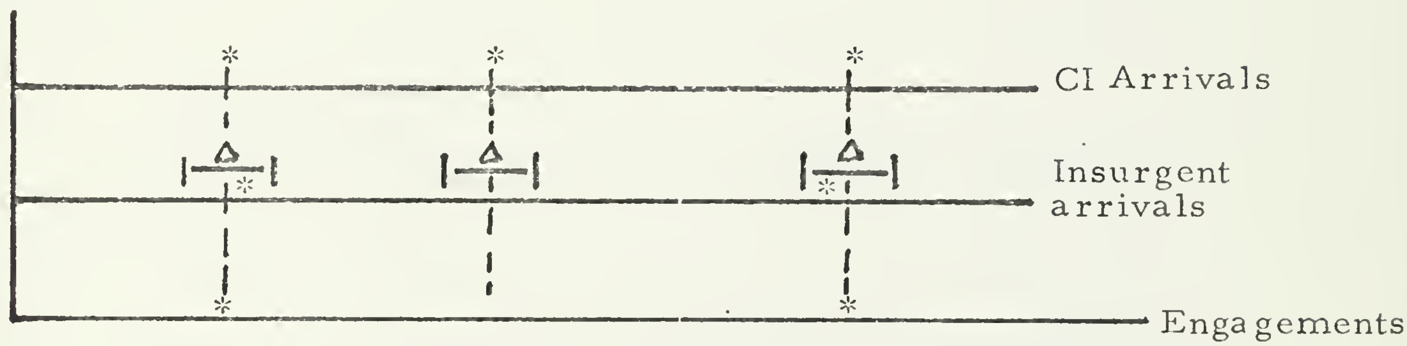
Figure 6.

It is assumed that CI patrol inter-arrival times, X_i, are distributed F_1 for X_1 and F for X_i for all $i \geq 2$ with rate L_i for the i^{th} arc. Insurgent infiltration group inter-arrival times, Y_i, are distributed G_1 for i=1, and G for all $i \geq 2$, with rate L_j for the j^{th} route segment. Define Δ to be a region about the trail intersection, in units of time, beyond which it is impossible for a CI patrol located at the intersection to identify the presence of an insurgent group on the trail.

Accordingly, it is assumed that if at least one CI patrol is located at the trail intersection, while simultaneously at least one insurgent group is located within the critical zone defined by Δ, then an engagement between forces will occur with probability 1. It is

further assumed that due to the values normally associated with L_i, L_j and Δ, the probability that two or more insurgent elements are within Δ at any given time is zero, as is the probability of two or more CI patrols arriving at the trail intersection during any interval Δ.

The occurrence of engagements between CI and insurgent forces at the trail intersection forms a stochastic renewal process. A hypothetical realization of this process, which obviously depends on the arrival of CI and insurgent elements, appears in Figure 7. It should be noted that the arrival of CI patrols, as well as the arrival of insurgent infiltration groups also define stochastic renewal processes.



REALIZATIONS OF STOCHASTIC RENEWAL PROCESSES

Figure 7.

In Figure 7, the * indicate the occurrence of a renewal, either in the form of an arrival or an engagement. If a CI patrol arrives at the trail intersection at some random time t, and an insurgent renewal occurs during the interval $(t-\Delta/2, t+\Delta/2)$, then an engagement occurs with probability 1. As was stated previously,

X_1 distributed F_1; X_n distributed F for $n \geq 2$

Y_1 distributed G_1; Y_n distributed G for $n \geq 2$

Z_i distributed H for all i

where Z_i is the time between the i^{th} and $(i-1)^{st}$ engagement.

Define: $M_x(t)$: the expected number of CI patrols to cross the trail intersection in any interval of length t, under a general renewal process.

$M_y(t)$: the expected number of insurgent groups to enter the critical zone during any interval of length t, under a general renewal process.

$M_z(t)$: the expected number of engagements to occur at the trail intersection during any interval of length t, under a general renewal process.

Obviously then,

$$M_z(t) = p\, M_x(t)$$

where p is the probability that a CI patrol crossing the trail intersection will become involved in an engagement with an insurgent element. That is, each CI patrol crossing the trail intersection represents a Bernoulli trail with the probability of being involved in an engagement, a success, p. The sum of these Bernoulli random variables is Binomially distributed with parameters p and $N_x(t)$, where $N_x(t)$ is the actual number of CI patrols to cross the trail intersection during any interval of length t. If $N_z(t)$ is defined as the actual number of engagements occurring in any interval of length t, then

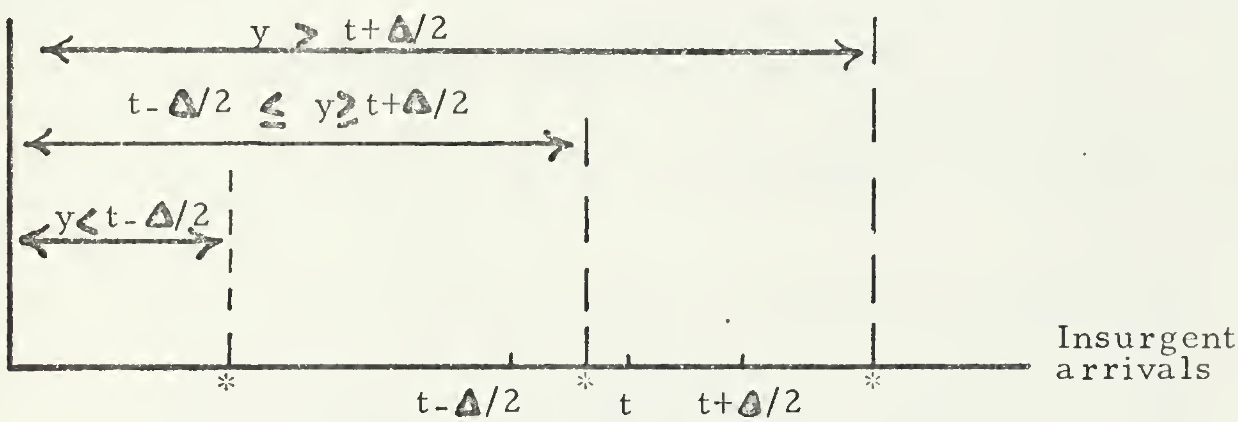
$$N_z(t) = p\, N_x(t)$$

Taking expected values of both sides of the above expression, results in the expression

$$M_z(t) = p\, M_x(t)$$

For this relationship to be of any significance in estimating the parameter of the infiltration distribution, it will be necessary to develop an expression for the Binomial parameter p.

To develop a useful expression for the parameter p, assume that at some arbitrary time t, a CI patrol reaches the trail intersection described in Figure 6. The parameter p then is the probability that this particular patrol will become involved in an engagement with one or more insurgent elements. Figure 8 below describes such a situation.



REALIZATION OF INSURGENT ARRIVALS

Figure 8.

Again, the * represent the arrival of insurgent infiltration elements. To develop an expression for p, it will be helpful to condition on the arrival of the first insurgent group. That is,

$$\begin{aligned} p/Y_1 = y \;&=\; 0 && y \quad t+ \quad /2 \\ &=\; 1 && t-\Delta/2 \leq y \leq t+\Delta/2 \\ &=\; P_o(t-y) && y < t-\Delta/2 \end{aligned}$$

where $P_o(t-y)$ is the probability of an engagement, that is an insurgent arrival during $(t-\Delta/2, t+\Delta/2)$, under an ordinary renewal process with new origin at $(t-y)$. To determine p, it is necessary to uncondition the conditional probability expression p/Y_1. That is,

$$p = E(p/Y_1) = \int_o^{t-\Delta/2} P_o(t-y)dG_1(y) + \int_{t-\Delta/2}^{t+\Delta/2} d.G_1(y)$$

or

$$p = P_o(t-y)*G_1(y) + G_1(t+\Delta/2) - G_1(t-\Delta/2)$$

where the operator * now indicates the convolution operation. For an ordinary renewal process, where $G_1=G$, the above expression for p is simply,

$$p_o = P_o(t-y)*G(y) + G(t+\Delta/2) - G(t-\Delta/2)$$

Using the Laplace-Stieltges Transform, $G^*(s) = \int_{-\infty}^{\infty} e^{-sX} dG(x)$, [3]it may be shown that,

$$p^*(s) = P_o^*(s)\, G_1^*(s) + \mathcal{L}\left\{G_1(t+\Delta/2) - G_1(t-\Delta/2);\ s\right\}$$

and $$p_o^*(s) = P_o^*(s)\, G^*(s) + \mathcal{L}\left\{G(t+\Delta/2) - G(t-\Delta/2)\ ;\ s\right\}$$

where $\mathcal{L}\{G(t);s\}$, denotes the Laplace-Stieltges Transform operation. Solving the preceeding two relationships simultaneously results in the following expression for $p^*(s)$.

$$p^*(s) = \mathcal{L}\left\{G(t+\Delta/2) - G(t-\Delta/2);s\right\} M^*(s) + \mathcal{L}\left\{G_1(t+\Delta/2) - G_1(t-\Delta/2);s\right\}$$

where $M^*(s)$ is the general renewal function for the process defined by insurgent arrivals, and [3]

$$M^*(s) = \frac{G_1^*(s)}{1 - G^*(s)}$$

Inverting the Laplace-Stieltges Transform, $P^*(s)$, to obtain an expression for p, results in

$$p = M_y(t) * [G(t+\Delta/2) - G(t-\Delta/2] + G_1(t+\Delta/2) - G_1(t-\Delta/2)$$

$$= \int_0^{t-\Delta/2} [G(t+\Delta/2-y) - G(t-\Delta/2-y]\, dM_y(y) + G_1(t+\Delta/2) - G_1(t-\Delta/2)$$

Re-defining the arguments for simplicity as follows,

$$t+\Delta/2 = t+\Delta$$

$$t-\Delta/2 = t$$

results in

$$p = \int_0^t [G(t+\Delta-y) - G(t-y)]\, dM_y(y) + G_1(t+\Delta) - G_1(t)$$

$$= \int_0^t G(t+\Delta-y)\, dM_y(y) - \int_0^t G(t-y) dM_y(y) + G_1(t+\Delta) - G_1(t)$$

However,

$$M(t) = G_1(t) + M_y(t) * G(t) \text{ and}$$

$$M_y(t) * G(t) = \int_0^t G(t-y)\, dM_y(y)$$

therefore,

$$p = \int_0^t G(t+\Delta-y) dM_y(y) + G_1(t+\Delta) - M_y(t)$$

$$= M_y(t+\Delta) - \int_t^{t+\Delta} G(t+\Delta-y) dM_y(y) - M_y(t)$$

$$= \int_t^{t+\Delta} [1-G(t+\Delta-y)]\, dM_y(y)$$

$$= \int_t^{t+\Delta} \overline{G}(t+\Delta-y)\, dM_y(y)$$

Now, assuming that the patrolling and infiltration processes have been operating for some extended period of time, the renewal processes

described may be considered an equilibrium processes. If the arrival of insurgent elements is an equilibrium process, then,

$$M_y(y) = \rho y$$

where ρ is the rate at which insurgent elements arrive at the trail intersection [4]. Therefore,

$$p = \int_t^{t+\Delta} \bar{G}(t+\Delta-y)\, d\rho y = \rho \int_t^{t+\Delta} \bar{G}(t+\Delta-y)\, dy$$

Let $X=t+\Delta-y$, and

$$p = \rho \int_{\Delta}^{0} \bar{G}(x)dx = \rho \int_0^{\Delta} \bar{G}(x)dx = Fe\,(\Delta)$$

where F_e is the equilibrium distribution for G, the insurgent inter-arrival distribution. If it is assumed that G is the exponential distribution with parameter L_j for the j^{th} infiltration route segment, then,

$$p = 1 - e^{-L_j \Delta}$$

Note, the probability of an engagement is dependent on Δ, not t.

Returning to the expression $M_z(t) = p\, M_x(t)$, it follows that,

$$M_z(t) = F_e\,(\Delta)\, M_x(t)$$

However, since the distribution of CI arrivals along the i^{th} patrol route is exponential with parameter L_i, determined from the solution to the linear programming problem described in II D above, it follows that

$$M_x(t) = L_i t$$

For large t, $M_z(t)$, the expected number of engagements at the intersection during any interval of length t, converges with the actual

number of engagements which occur during such a period t. Therefore, if the actual number of engagements in the time t is used to estimate $M_z(t)$, the only unknown remaining in the expression

$$M_z(t) = F_e(\Delta)\, M_x(t) = \left\{ i - e^{-L_j \Delta} \right\} L_i t$$

is the parameter L_j, the rate of infiltration along the j^{th} insurgent infiltration route segment.

It should be noted, that the estimate for L_j is very much dependent on the accuracy of the estimate for $M_z(t)$. The procedure discussed above assumes that a large enough period, t, exists such that the actual number of engagements which occur will converge with $M_z(t)$ and that during this period, the infiltration rate, L_j, remains essentially unchanged. Certainly, it must be recognized that if a significant number of engagements occur at the intersection under investigation, the insurgent may well alter the infiltration schedule which will possibly alter the appropriate L_j.

In view of the problems discussed in the previous paragraph, as well as the fact that due to the normal values associated with L_i, L_j and Δ, the actual number of engagements occurring in any reasonable period t will necessarily be quite small. A result of this situation will be a large variance associated with the estimators for $M_z(t)$, and hence L_j.

III. CONCLUSIONS

The model developed in Section II of this paper represents a useful mathematical tool which may be of significant practical value to military staff planners in an Internal Defense and Development environment. In this section, the objective will be to briefly discuss some of the various capabilities and limitations of the model.

A. CAPABILITIES

Conditioned on the validity of the assumptions discussed in Section II, the model developed is capable of the following:

1. Given a CI patrol network and various constraints which concern the frequency with which network arcs should be patrolled, the availability of patrol resources, and the conservation of patrol effort at each node in the network, the model will identify an efficient and effective patrol schedule for attaining the previously discussed CI objectives of minimizing patrol effort subject to the above mentioned constraints, while minimizing the insurgents ability to predict future CI operational plans.

2. Based on the network and patrol schedule developed in (1) above, the model is capable of predicting the expected number of CI patrols active in the network at any time t.

3. In addition, the model is capable of predicting the expected duration of a randomly selected CI patrol operating in the network discussed above.

4. Finally, the model is capable of estimating the rate of insurgent infiltration along a given infiltration route segment which is randomly surveilled by CI patrols.

B. LIMITATIONS

The model presented in this paper has been developed based on various assumptions. Obviously, the validity of any results obtained with this model is necessarily directly related to the validity of these assumptions based on the specific situation being investigated. For that reason, it is felt necessary to direct additional attention to these key assumptions at this time.

1. With regard to the method of operation attributed to the CI force, it was assumed that, in view of the nature of the CI forces under investigation, the mobile reconnaissance patrol was more appropriate than the previously discussed "trail-watcher" technique. This assumption was made based on the generally limited capability, both with respect to training and equipment, of the CI force being considered in this paper.

2. In developing the CI patrol plan, it was assumed that the ability of the insurgent to accurately predict future CI operational plans could be minimized through the use of a "memoryless" scheme.

It must be realized that there are actually several other means by which insurgent forces can predict such information other than past performance. Certainly, insurgent security forces located in the immediate vicinity of the CI camp provide information on the initial movement of CI patrols departing the camp. In addition, insurgent trail parties, assigned the task of shadowing CI patrols, are capable of reporting subsequent patrol movements. Finally, insurgent security elements stationed along frequently used infiltration routes provide advance warning to infiltrating groups of the presence of threatening CI patrols. Such techniques provide the insurgent with essential information which may have a significant impact on infiltration, and hence surveillance-interdiction operations. Since CI counter techniques have little effect on such insurgent activities, the model developed has not considered these aspects.

3. From the discussion in 2. above, it is obvious, that under certain circumstances the assumption of independence between the movement of CI patrols and insurgent infiltration elements within the TAOR may very well be invalid. Such an assumption was necessary in the development of the estimator for the infiltration rate along a given infiltration route segment.

IV. AREAS FOR FUTURE RESEARCH

The purpose of this section is to briefly describe some additional problems associated with the counter-insurgency scenario developed in section I. D of this paper, which are considered appropriate subjects for mathematical analysis.

A. ALLOCATION OF INTELLIGENCE EFFORT

Essentially, the problem is, given a Tactical Area of Responsibility (TAOR) such as that defined in Figure 1, with a finite number of possible insurgent base camp areas, how best to allocate the limited CI intelligence gathering resources (reconnaissance patrols) to maximize the percentage of the insurgent force located by the CI search effort.

B. CAPTURING AN INSURGENT HEADQUARTERS

The problem is somewhat similar to that described in A above. However, in this situation the objective is to capture the insurgent command element, which is normally co-located with a subordinate unit for security purposes. The problem is to identify the CI optimal strategy to be used to maximize the probability of capturing the insurgent headquarters element. It is assumed that the headquarters element will be located in each of the various possible base camp areas with some probability, depending on various factors. In addition, one must consider the fact that simply concentrating a

superior force at the proper locations, with adequate surprise, may not be sufficient to insure the capture of the insurgent headquarters element. In many circumstances, such an insurgent commander will use the security force as a decoy, or rear guard, while the command element escapes from the battle area.

C. THE CHECKPOINT PROBLEM

Assume a CI security checkpoint has been established at some point along a major trail or road within the assumed TAOR. In addition to the authorized traffic which passes through this checkpoint, the insurgent force attempts to use the road for the movement of men, material and intelligence agents. The CI security force can stop and check all persons and vehicles passing through the checkpoint, but only at great expense to the government and inconvenience to the population. If the CI force were to adopt this policy, the insurgent would simply by-pass the checkpoint, using some cross-country route, at some cost to the insurgent (time and increased losses). The problem then is to identify the optimal search policy to be used by the CI security force, which will maximize the probability of intercepting an insurgent element attempting to pass through the checkpoint. Such a policy must not force the insurgent to adopt the cross-country alternative to the checkpoint, and must take into consideration resource constraints which restrict CI policy.

D. INSURGENT GROWTH MODEL

The objective here is to develop a model which will identify changes in the level of insurgent activity, as well as changes in insurgent tactics. It is assumed that the basis of the model would in some way depend on those political, military, social and economic factors which form the necessary ingredients for any insurgency. Given accurate, up-to-date data representing these various factors, the model should be capable of using this input to determine whether or not there has been a significant change in the level of insurgent activity. If such a change is indicated, the model should be capable of identifying those areas, or factors, which represent primary insurgent targets. A model capable of detecting and identifying insurgent trends, directions or intentions would obviously be of great benefit to a CI force.

E. THE SURVEILLANCE-INTERDICTION PROBLEM

The model developed in Section II of this paper is generally based on a pessimistic attitude concerning the capabilities of military/para-military forces of the type normally associated with the remote camps previously discussed. If on the other hand, one was to develop a more optimistic attitude, one which reflected a CI capability to effectively surveil and interdict insurgent lines of communication by aggressively closing with the enemy in combat, the strategy developed would most likely differ from that thus far presented. The problem in

this new situation would be the allocation of limited resources to effectively impede the flow of men and supplies through a given trail network. The solution should reflect both the employment of small scale reconnaissance elements to fix insurgent forces, and the employment of larger combat forces designed to destroy the infiltration once it has been identified.

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13. ABSTRACT

This thesis presents a mathematical model of conducting limited scale surveillance-interdiction operations in a remote area environment. The model, which is stochastic in nature, provides CI patrol schedules in such a manner as to minimize the patrol effort expended, subject to various resource and operational constraints established by the local CI commander. Patrol schedules are developed so as to minimize the ability of the insurgent force to accurately predict future CI operational plans. In addition to generating patrol schedules, for a specified patrol plan, the model predicts additional information such as the expected number of active CI patrols at any given time, the expected duration of a CI patrol, as well as an estimate of the insurgent infiltration rate along a given segment of the infiltration route. Finally, the model is capable of providing probabilistic statements concerning the adequacy of a specified force level to satisfy the requirements of a given patrol plan.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
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Surveillance						
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